Near-Tight Approximation Algorithms for Bottleneck Multiple Knapsack Problems

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Abstract

In the bottleneck multiple knapsack problem (BMKP), we are given a set of items and a set of knapsacks, where each item has a profit and a weight, and each knapsack has an identical capacity. Our goal is to assign items to knapsacks so as to maximize the minimum profit received by any knapsack subject to the capacity constraints.

In the special case that the item profit and its weight are equal, known as the *Bottleneck Multiple Subset Sum* problem, Caprara, Kellerer, and Pferschy [1] gave a $\frac{2}{3}$-approximation algorithm and proved that achieving a $(\frac{2}{3}+\varepsilon)$-approximation for any $\varepsilon > 0$ is impossible unless $P = NP$. More recently, Hummel [2] studied maximin share guarantees in a hereditary set system, which generalizes the setting of BMKP. Their result implies a $\frac{2}{5}$-approximation for BMKP. This left a substantial gap between the known hardness threshold of $\frac{2}{3}+\varepsilon$ and the algorithmic guarantee of $\frac{2}{5}$.

Main Results. We first give a nearly tight approximation guarantee for BMKP. Namely, we obtain a $(\frac{2}{3}-\varepsilon)$-approximation algorithm, which almost nearly matches the $(\frac{2}{3}+\varepsilon)$ inapproximability barrier of Caprara et al. [1].

Theorem 1. *For any constant $\varepsilon > 0$, there is a polynomial-time $(\frac{2}{3}-\varepsilon)$-approximation algorithm for the bottleneck multiple knapsack problem with running time $n^{2^{O(1/\varepsilon \log(1/\varepsilon))}}$, where n is the number of items.*

We further consider a natural generalization where the knapsack capacities are non-identical. Again a nearly tight approximation guarantee is established. The ratio is apparently different from that of BMKP.

Theorem 2. *For any constant $\varepsilon > 0$, there is a polynomial-time $(\frac{1}{2}-\varepsilon)$-approximation algorithm for the bottleneck multiple knapsack problem with running time $2^{2^{\mathrm{poly}(1/\varepsilon)}} \cdot \mathrm{poly}(n)$.*

Theorem 3. *Unless $P = NP$, the bottleneck multiple knapsack problem has no polynomial-time $(\frac{1}{2}+\varepsilon)$-approximation algorithm for any constant $\varepsilon > 0$.*

References

[1] A. Caprara, H. Kellerer, and U. Pferschy. The multiple subset sum problem. *SIAM Journal on Optimization*, 11(2):308–319, 2000.

[2] H. Hummel. Maximin shares in hereditary set systems. *ACM Transactions on Economics and Computation*, 13(3):1–33, 2025.